EXTENSION OF THE CORRELATION COEFFICIENT FOR INTERVAL-VALUED ATANASSOV'S INTUITIONISTIC FUZZY LOGIC

ALEX BERTEI[1]; LUCIANA FOSS[1]; RENATA REISER[1]

[1]Federal University of Pelotas (UFPEL) – {abertei, lfoss, reiser} @inf.ufpel.edu.br

1. INTRODUCTION

The fuzzy set (FS) theory has been widely used in each field of modern society since it was proposed by ZADEH (1965). In the FS theory only membership values are assigned to each element lying in between the two limits of the unit interval.

Atanassov's intuitionistic fuzzy sets (A-IFSs) (ATANASSOV, 1986) are used to explain the situations where there is hesitancy in the information describing the membership and non-membership degrees of elements in FSs. In this context, the concept of an A-IFS considers the information of both the membership degree and the non-membership degree.

Further, ATANASSOV; GARGOV (1989) presented the concept of the Atanassov's interval-valued intuitionistic fuzzy set (A-IVIFS), denoting the membership degree and the non-membership degree by the closed subinterval of the unit interval [0, 1] and effectively extended the A-IFS's capability of handling the uncertain information.

The approach of A-IFS leads to a great and relevant number of studies. See, e.g., the Atanassov's intuitionistic fuzzy index of an element remaining the hesitation between the membership and non-membership degree dealing with similarity measure:

(a) in SZMIDT; KACPRZYK (2007) analyzing the consensus of an expert preference in a group decision making (DM);

(b) in GONZALEZ-ARTEAGA et al. (2016), indicating the similar degree of A-IFS; and

(c) in SZMIDT; KACPRZYK (2001), interpreting the entropy of A-IFSs and describing its fuzziness degree.

All of them are closely connected with the correlation coefficient (A-CC) between two IFSs, which is able to express whole expert systems in fuzzy reasoning, mainly those applied to decision-making (DM) processes. A-CC should provide an expression given by real parameters from -1, as the most negative (decreasing) linear relationship to 1, as the most positive (increasing) linear relationship. So, the closer an A-CC is to either -1 or 1, the stronger the correlation.

The correlation coefficient is applied many research areas, such as digital image processing, clustering analysis, decision making, medical diagnosis and pattern recognition.

In BUSTINCE; BURILLO (1995) the authors introduced the concepts of correlation coefficient of interval-valued intuitionistic fuzzy sets studying their first properties. XU (2006) proposed a new method for deriving an A-CC of the IFS and also extended it to the A-IVIFS theory.

Multiple attribute group decision-making (MADM) methods using the entropy weights-based A-CC for IFSs and IVIFSs are proposed by YE (2013) to identify the best alternative in DM problems.

The interval-valued intuitionistic fuzzy approaches in LIU (2016) combines weighted averaging geometric operators with the entropy theory to calculate reasonable criteria weight.

Moreover, the A-CC of interval-valued intuitionistic fuzzy numbers considering its uncertainty is revised. The work of REISER; BEDREGAL (2017) focusses on special classes of interval-valued Atanassov's intuitionistic fuzzy implications related to R-, S-, QL- and D- implication classes, in order to analyze the A-CC in their dual and conjugate constructions. As the main contribution, the concept of A-CC related to fuzzy connectives in Atanassov's Interval-Valued Intuitionistic Fuzzy Logic (A-IVIFL) are aplied to conjugate and their dual constructions.

As the main objective, this article extends the A-CC formula studied in BERTEI; REISER (2019) from A-IFS to A-IVIFS.

2. METHODOLOGY

In order to achieve the first partial results, initially we studied the most main research-related topics and relevant papers in the literature where realized a bibliographic review which involve the FS, A-IFS, A-IVIFS and the correlation coefficient. We have also analyzed their relationships whit their respective sets, reporting the axiomatic fuzzy negations and also characterizing the fuzzy connectives related to intersections and unions, the triangular norms and conorms, respectively.

In previous works by BERTEI; REISER (2019) presented a formula for the correlation coefficient for A-IFS. The paper studies the correlation coefficient related to the Atanassov's intuitionistic fuzzy sets which are obtained as image of modal operators. Extended results from the action of A-CC over necessity and possibility modal operators are considered, determining the A-CC of A-IFS obtained as image of the !A and ?A modal level operators and discussing the main conditions under which the main properties related to such fuzzy sets are preserved by conjugate and complement operations. In addition, a simulation based on the proposal methodology using modal level operators is applied to a medical diagnosis analysis.

Atanassov and Gargov [Atanassov and Gargov 1989] made great efforts on extending the A-IFS [Atanassov 1986] to the A-IVIFS, and developed basic operations on A-IVIFS. An A-IVIFS $\mathbb{A}_I$ in a universe $\chi \neq \emptyset$ is expressed as follows:

$$\mathbb{A}_I = \{(x, \mu_{\mathbb{A}_I}(x), \nu_{\mathbb{A}_I}(x)) : x \in \chi, \mu_{\mathbb{A}_I}(x) + \nu_{\mathbb{A}_I}(x) \leq_{\mathbb{U}} \mathbf{1}\}, \qquad (1)$$

and the set of all A-IVIFS is denoted by $\mathcal{A}_I$. The $\mu_{\mathbb{A}_I}(x)$, $\nu_{\mathbb{A}_I}(x)$ and $\pi_{\mathbb{A}_I}(x)$ are respectively, its interval-valued membership and non-membership degrees and Atanassov's Interval-valued Intuitionistic Fuzzy Index in the A-IVIFS $\mathbb{A}_I$. So, for each $x \in \chi$, $\mu_{\mathbb{A}_I}(x)$ and $\nu_{\mathbb{A}_I}(x)$ are closed subintervals in the unitary interval [0,1] and their lower and upper endpoints are expressed as follows:

$$\underline{\mu_{\mathbb{A}_I}(x)} = \underline{X_1}, \overline{\mu_{\mathbb{A}_I}(x)} = \overline{X_1} \ \ and \ \ \underline{\nu_{\mathbb{A}_I}(x)} = \underline{X_2}, \overline{\nu_{\mathbb{A}_I}(x)} = \overline{X_2}.$$

Thus, we can replace Eq. (1) as

$$\mathbb{A}_I = \left\{\left(x, \left[\underline{\mu_{\mathbb{A}_I}(x)}, \overline{\mu_{\mathbb{A}_I}(x)}\right], \left[\underline{\nu_{\mathbb{A}_I}(x)}, \overline{\nu_{\mathbb{A}_I}(x)}\right]\right) : x \in \chi, 0 \leq \overline{\mu_{\mathbb{A}_I}(x)} + \overline{\nu_{\mathbb{A}_I}(x)} \leq 1\right\} \qquad (2)$$

3. RESULTS AND DISCUSSION

The correlation coefficient of the article by BERTEI; REISER (2019) has been extended to Interval-Valued Atanassov's Intuitionistic Fuzzy Logic, where the new correlation coefficient will take into account the interval values for membership and non-membership, and for each of these there will be a minimum interval value and supreme. For that, let $\chi \neq \emptyset$ such that $X = \{x_1, x_2, \dots, x_n\}$ and $\mathbb{A}_I = \{(x, \mu_{\mathbb{A}_I}(x), \nu_{\mathbb{A}_I}(x)) : x \in X\}$ be an A-IVIFS. For all $x_i \in X$, $i \in \mathbb{N}_n$, consider the following notation related to the Atanassov's interval-valued intuitionistic fuzzy values of an A-IVIFS $\mathbb{A}_I$:

$$\left(\left[\underline{\mu_{\mathbb{A}_I}(x_i)}, \overline{\mu_{\mathbb{A}_I}(x_i)}\right], \left[\underline{\nu_{\mathbb{A}_I}(x_i)}, \overline{\nu_{\mathbb{A}_I}(x_i)}\right], \left[\underline{\pi_{\mathbb{A}_I}(x_i)}, \overline{\pi_{\mathbb{A}_I}(x_i)}\right]\right) = \left(\left[\underline{X_{i1}}, \overline{X_{i1}}\right], \left[\underline{X_{i2}}, \overline{X_{i2}}\right], \left[\underline{X_{i3}}, \overline{X_{i3}}\right]\right).$$

Definition 1. Let $\mathbb{A}_I$ and $\mathbb{B}_I$ be A-IVIFS. The correlation coefficient between $\mathbb{A}_I$ and $\mathbb{B}_I$ is the operator $C_I : \mathcal{A}_I \times \mathcal{A}_I \to \mathbb{U}$ satisfying the following properties:

I. $C_I(\mathbb{A}_I, \mathbb{B}_I) = C_I(\mathbb{B}_I, \mathbb{A}_I)$;
II. If $\mathbb{A}_I = \mathbb{B}_I$ then $C_I(\mathbb{A}_I, \mathbb{B}_I) = \mathbf{1}$;
III. $-\mathbf{1} \leq C_I(\mathbb{A}_I, \mathbb{B}_I) \leq \mathbf{1}$.

For all $X_{ik} = \left[\underline{X_{ik}}, \overline{X_{ik}}\right]$, $i \in \mathbb{N}_n, k \in \mathbb{N}_3$, consider the following denotation

$$(\mu_{\mathbb{A}_I}(x_1), \mu_{\mathbb{A}_I}(x_2), \dots, \mu_{\mathbb{A}_I}(x_n)) = (X_{11}, X_{21}, \dots, X_{n1}) = \vec{X}_1;$$
$$(\nu_{\mathbb{A}_I}(x_1), \nu_{\mathbb{A}_I}(x_2), \dots, \nu_{\mathbb{A}_I}(x_n)) = (X_{12}, X_{22}, \dots, X_{n2}) = \vec{X}_2;$$
$$(\pi_{\mathbb{A}_I}(x_1), \pi_{\mathbb{A}_I}(x_2), \dots, \pi_{\mathbb{A}_I}(x_n)) = (X_{13}, X_{23}, \dots, X_{n3}) = \vec{X}_3.$$

The vectors $\vec{X}_1, \vec{X}_2, \vec{X}_3 \in \widetilde{\mathbb{U}}^n$ represent the membership, non-membership and hesitant degrees of all $x_i \in X$, for $i \in \mathbb{N}_n$.

Proposition 1. Let $\mathbb{M}$ be the interval-valued arithmetic mean. For all $\mathbb{A}_I, \mathbb{B}_I \in \mathcal{A}_I$, the function $C_I : \mathcal{A}_I \times \mathcal{A}_I \to \mathbb{U}$ given as follows:

$$C_I(\mathbb{A}_I, \mathbb{B}_I) = \frac{1}{3}(C_1(\mathbb{A}_I, \mathbb{B}_I) + C_2(\mathbb{A}_I, \mathbb{B}_I) + C_3(\mathbb{A}_I, \mathbb{B}_I)) \quad (3)$$

when for all $k \in \mathbb{N}_3$, $C_k : \mathcal{A}_I \times \mathcal{A}_I \to \mathbb{U}$ is defined as follows:

$$C_k(\mathbb{A}_I, \mathbb{B}_I) = \frac{\sum_{i=1}^{n} (X_{ik} - \mathbb{M}(\vec{X}_k))(Y_{ik} - \mathbb{M}(\vec{Y}_k))}{\sqrt{\sum_{i=1}^{n} (X_{ik} - \mathbb{M}(\vec{X}_k))^2 \sum_{i=1}^{n} (Y_{ik} - \mathbb{M}(\vec{Y}_k))^2}} \quad (4)$$

is an interval-valued intuitionistic **correlation coefficient** ($\mathbb{CC}_I$) between $\mathbb{B}_I$ and $\mathbb{A}_I$.

Corollary 1. Let $C_I : \mathcal{A}_I \times \mathcal{A}_I \to \mathbb{U}$ be the interval-valued intuitionistic correlation coefficient between $\mathbb{A}_I$ and $\mathbb{B}_I$ defined by operator in Eq.(3). For all $k \in \mathbb{N}_3$, the operator $\mathbb{CC}_I$ $C_k : \mathcal{A}_I \times \mathcal{A}_I \to \mathbb{U}$ can be defined as follows:

$$C_k(\mathbb{A}_I, \mathbb{B}_I) = \frac{\sum_{i=1}^{n} \left(X_{ik} - \frac{1}{n}\sum_{j=1}^{n} X_{jk}\right)\left(Y_{ik} - \frac{1}{n}\sum_{j=1}^{n} Y_{jk}\right)}{\sqrt{\sum_{i=1}^{n} \left(X_{ik} - \frac{1}{n}\sum_{j=1}^{n} X_{jk}\right)^2 \sum_{i=1}^{n} \left(Y_{ik} - \frac{1}{n}\sum_{j=1}^{n} Y_{jk}\right)^2}} \quad (5)$$

Corollary 2. Let $C_I : \mathcal{A}_I \times \mathcal{A}_I \to \mathbb{U}$ be the interval-valued intuitionistic correlation coefficient between $\mathbb{A}_I$ and $\mathbb{B}_I$ defined by operator in Eq.(3). For $k \in \mathbb{N}_3$, the operator C_k in Eq.(5) can also be expressed as follows:

$$\underline{C_k(\mathbb{A}, \mathbb{B})} = \frac{\sum_{i=1}^{n} \left(\underline{X_{ik}} - \frac{1}{n}\sum_{j=1}^{n} \overline{X_{jk}}\right)\left(\underline{Y_{ik}} - \frac{1}{n}\sum_{j=1}^{n} \overline{Y_{jk}}\right)}{\sqrt{\sum_{i=1}^{n} \left(\underline{X_{ik}} - \frac{1}{n}\sum_{j=1}^{n} \overline{X_{jk}}\right)^2 \sum_{i=k}^{n} \left(\underline{Y_{ik}} - \frac{1}{n}\sum_{j=1}^{n} \overline{Y_{jk}}\right)^2}}$$

$$\overline{C_k(\mathbb{A},\mathbb{B})} = \frac{\sum_{i=1}^{n}\left(\overline{X_{ik}}-\frac{1}{n}\sum_{j=1}^{n}\underline{X_{jk}}\right)\left(\overline{Y_{ik}}-\frac{1}{n}\sum_{j=1}^{n}\underline{Y_{jk}}\right)}{\sqrt{\sum_{i=1}^{n}\left(\overline{X_{ik}}-\frac{1}{n}\sum_{j=1}^{n}\underline{X_{jk}}\right)^2\sum_{i=1}^{n}\left(\overline{Y_{ik}}-\frac{1}{n}\sum_{j=1}^{n}\underline{Y_{jk}}\right)^2}}$$

4. CONCLUSIONS

In this paper the formulation of correlation coefficient was introduced, this formulation has been applied to an interval-valued Atanassov's intuitionist fuzzy set.

Further work intends to extend these correlation formulations of A-IVIFSs to fuzzy connectives and modal operators. Moreover, consider action of strong Atanassov's Interval-Valued Intuitionistic Fuzzy Negation in order to verify the conditions under which the A-CC related to such A-IVIFS are obtained. And, also apply this new formulation correlation to a set of input data and develop an application for the problems of decision making.

5. BIBLIOGRAPHIC REFERENCES

ATANASSOV, K. T. Intuitionistic fuzzy sets. **Fuzzy Sets and Systems**, Amsterdam, v.20, n.1, p.87 - 96, 1986.

ATANASSOV, K.; GARGOV, G. Interval Valued Intuitionistic Fuzzy Sets. **Fuzzy Sets Systems**, Amsterdam, v.31, n.3, p.343-349, 1989.

BERTEI, A.; REISER, R. Correlation coefficient of modal level operators: An application to medical diagnosis. In: **11TH INTERNATIONAL JOINT CONFERENCE ON COMPUTATIONAL INTELLIGENCE (FCTA 2019)**, Vienna 2019, Anais International Joint Conference on Computational Intelligence, 2019.v.1. p.1-9.

SZMIDT, E.; KACPRZYK, J. A new similarity measure for intuitionistic fuzzy sets: Straightforward approaches may not work. In **IEEE INTERNATIONAL CONFERENCE ON FUZZY SYSTEMS**, 2007, p. 1–6.

GONZALEZ-ARTEAGA, T.; DE ANDRÉS CALLE, R.; CHICLANA, F. A new measure of consensus with reciprocal preference relations: The correlation consensus degree**. Knowledge-Based Systems**, v.107, p.104 – 116, 2016.

SZMIDT, E.; KACPRZYK, J. Entropy for intuitionistic fuzzy sets. **Fuzzy Sets and Systems**, v.118, n.3, p.467-477, 2001

BUSTINCE, H.; BURILLO, P. Correlation of interval-valued intuitionistic fuzzy sets. **Fuzzy Sets and Systems**, v.74, n.2, p.237-244, 1995

XU, Z. On correlation measures of intuitionistic fuzzy sets. In **INTELLIGENT DATA ENGINEERING AND AUTOMATED LEARNING – IDEAL 2006**, Berlin, 2006 Anais Springer Berlin Heidelberg, 2006. p.16-24.

YE, J. Multiple attribute group decision-making methods with completely unknown weights in intuitionistic fuzzy setting and interval-valued intuitionistic fuzzy setting. **Group Decision and Negotiation**, v.22, n.2, p.173-188, 2013.